Motivation and Ultraviolet Catastrophe - Quantum Mechanics

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Before the 20th century, all of the physics I've talked about up to this point worked incredibly well. However, around the turn of the century, tons of issues were bubbling up to the surface. Experimental physicists were designing experiments where particles acted like waves, light acted like particles which acted like waves, and much more. These tests were starting to shake up the foundations that been established for more than three centuries, leaving theoretical physicists scrambling for answers. This noteset describes the exact breaks in the foundations, and how the new field of quantum mechanics came to the rescue.

Issue 1: Blackbody Radiation

What is a Blackbody?

A blackbody is a piece of material that radiates light corresponding to its temperature, while also absorbing and reflecting light from its surroundings. Lots of things can be blackbodies. For example, if you've ever seen a blacksmith heating metal, it glows red, yellow, or white hot depending on its temperature. The glowing happens because the electrons on the surface move faster with higher heat, and "dance" the extra energy out by emitting light.

A perfect blackbody is an object that absorbs all the light that hits it,

Figure 1: Molten metal at a foundry glowing like a blackbody. Image credit to Enlightening Images on Unsplash.

hence the name “black” body. Physicists wanted to study the light a blackbody emitted, but ran into a big problem: how the hell do you make something that absorbs all the light that hits it? Physicists were clever, and created a big, hollow sphere with a hole in the side (pictured below), so that any light sent through the hole would bounce around and (mostly) get stuck inside. If you heat up the sphere, the hole would begin to glow! Thus, physicists got to work studying the light coming out of the hole.

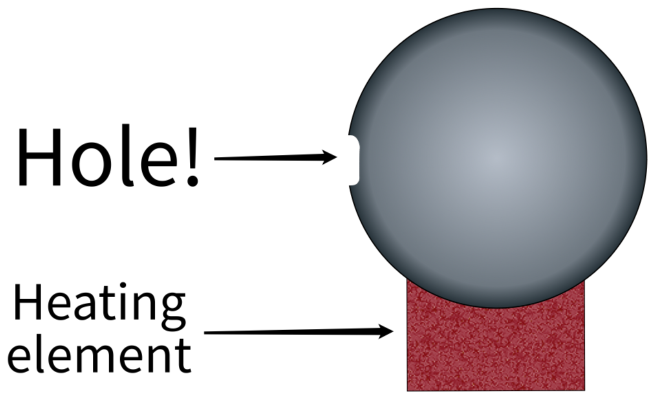

Figure 2: My attempt at drawing an experimental blackbody with a little heating element at the bottom.

Attempt 1: Wien’s Law

The first major attempt to describe the light came from Wilhelm Wien in 1889, which took the form:

$$I(\nu, T) = \alpha \nu^3 e^{-\frac{\beta \nu}{T}} \tag{1}$$

In this equation, $I(\nu, T)$ is the intensity distribution of the light spectrum (energy density of the emitted light as a function of frequency ν) of a blackbody at temperature T. Meanwhile, α and β are constants which can be adjusted via experiments. Although this formula worked well for high frequencies, things began to break down at lower frequencies. something else was needed...

Attempt 2: Rayleigh-Jeans Law

The next major attempt to hit the presses was the Rayleigh-Jeans Law, proposed in a sliding range of dates from 1900 to 1905 by Lord Rayleigh and James Jeans. They proposed the following law:

$$I(\nu, T) = \frac{2\pi \nu^4 kT}{c^3} \tag{2}$$

All the repeat variables are the same, with the additions of Boltzmann's constant ($k = 1.3807 * 10^{-23} \frac{J}{K}$) and the speed of light $c = 3 * 10^8 \frac{m}{s}$. The Rayleigh-Jeans law had the reserve problem of Wien's law: it worked GREAT at low frequencies, and completely blew up at high ones, starting in the ultraviolet frequency range. Because of this, inability for classical physics to explain what was going on was called the ***ultraviolet catastrophe***. Not good!

Attempt 3: Planck's Law

Now, we get to where quantum mechanics saves the day, and also explains part of what makes quantum mechanics so strange. In the early 1900's, the German physicist Max Planck proposed a solution that, at the time, seemed insane: what if the amount of energy that a light wave could give a piece of matter when they touched, or take away when sent out, could only be certain values? This flew in the face of classical physics intuition, which said that the energy a light wave could exchange with matter was continuous. Planck ignored these suggestions and suggested that the energy of the light emitted from the walls of the blackbody cavity only came out in integer multiples like this (with h as a constant):

$$E = nh\nu, \text{ where } n = 0, 1, 2, 3... \tag{3}$$

With this intuition, he then changed the Rayleigh-Jeans law to look like this:

$$I(\nu, T) = \frac{2\pi h\nu^5}{c^3(e^{\frac{h\nu}{kT}} - 1)} \tag{4}$$

To the astonishment of the entire physics community, this equation *EXACTLY* describes the spectrum of a blackbody at all frequencies. This meant that the light waves the matter in the blackbody couldn't spray out at just ANY energy. Not at all! It could only come out at specific, *quantized* energy levels. This also meant that the energy the electrons had to "dance" was also quantized into specific energies which, Planck hypothesized, were integer multiples of $h\nu$, just like equation 3!

Equation 4 became known as Planck's quantization rule, and h became Planck's constant ($h = 6.626 * 10^{-34}$ Js). With the idea that electrons and light could only tango at quantized energy levels, the revolution of quantum physics was on!

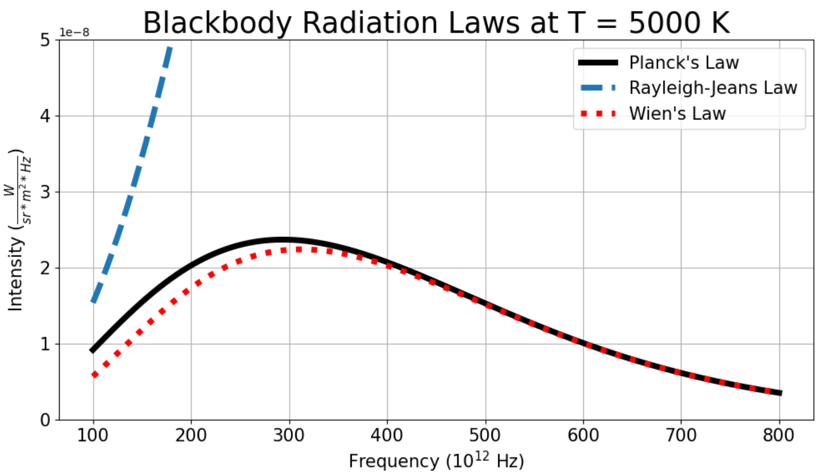

Figure 3: A graph of all three laws stacked against each other, with Wien's law having $\alpha = \frac{h}{c^2}$ and $b = \frac{h}{k}$. Note that the peak of Wien's law actually predicts the peak of Planck's law, expressed via $\nu_{\text{peak}} = \frac{b}{T}$.

Issue 2: The Photoelectric Effect

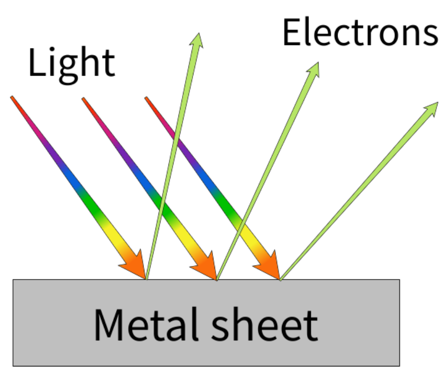

Figure 4: My attempt at drawing a photoelectric effect setup.

The next big issue with classical physics was the photoelectric effect. When you shine a light on a metal, electrons pop off. This is because they absorb enough energy to break free of the metal's surface. According to classical physics, with light waves that could exchange any amount of energy it wanted with the metal, the electrons should slowly soak up the light and then, when they have enough energy, fly away. If you shine more light onto the metal, the electrons leave with more energy. If you shine too little light on the metal, the electrons will have tons of trouble leaving.

This is, unsurprisingly, not what happened at all. As soon as someone shone a light on the metal, electrons were ejected instantly. No matter how weak the light was, no matter what direction they shone the light from, electrons were always ejected instantly. On top of that, if you shone a light with a frequency below a certain threshold, no electrons were emitted at all, no matter how long you waited. What the hell is up with that?!?

Einstein, who was having a fantastic year publishing stuff like special relativity, decided to take on the challenge. Emboldened by Planck's wacky idea, he proposed one even wackier: what if not only electron oscillation energies were quantized, but so was light? These "photons" would have discrete amounts of energy, packaged into a particle. This meant that not only does light act like a wave, as suggested by classical physics, but also a particle!

He predicted that when photons hit the metal surface, they interact with the electrons, which completely absorbs the photon. If the energy of the photon ($h\nu$) is greater than the work function ($W = h\nu_{\text{min}}$, the minimum amount of energy needed to eject a free electron), then the electron will fly off! The kinetic energy (K) of the ejected electron will then be:

$$h\nu = W + K \implies K = h\nu - W \implies K = h(\nu - \nu_{\text{min}}) \tag{5}$$

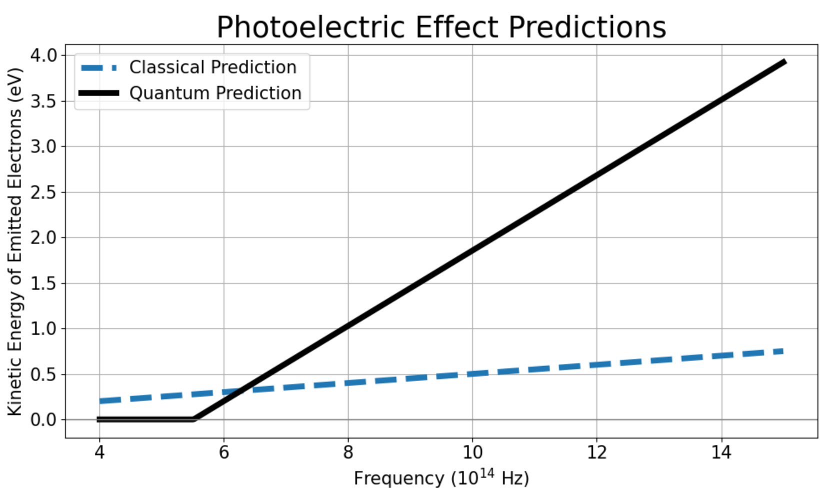

Figure 5: The graph of the classical vs quantum photoelectric effect. Once the photons get above a certain threshold frequency $\nu_0 = \nu_{\text{min}}$, electrons will start flying away!

This perfectly explained the photoelectric effect! Now, the foundations were REALLY coming down! Surely no other major changes would happen...

Issue 3: The Compton Effect

At the same time this theory of light as a particle was being postulated, the American physicist Arthur Compton (in Wooster, Ohio!!) was experimentally proving it. He set up an experiment where he would send light to scatter off electrons. Incident light comes in with a wavelength λ, hits the resting electron, and scatters away at an angle θ.

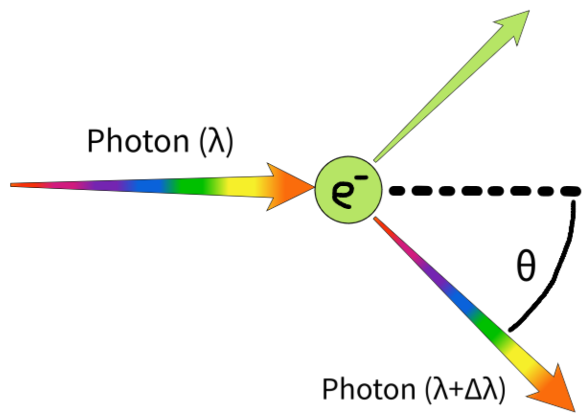

Figure 6: A drawing of the Compton effect. Light (a photon) hits an at-rest electron, elastically collides, and comes off with a shifted wavelength and at an angle.

Classically, the electron should have absorbed the light, jiggled and oscillated a bit, and then emitted it with the *same* wavelength as before. However, the wavelength changes! The light was coming out with a wavelength $\lambda + \Delta\lambda$, where the wavelength shift $\Delta\lambda$ depended on the scattering angle θ. Compton could only explain his results by saying he was dealing with *two* particles, not a wave and a particle. He said that the electron and the "photon" had to have collided elastically, conserving total energy and momentum (saying $E = h\nu$ and $p = \frac{E}{c}$), perfectly predicting the relation between $\Delta\lambda$ and θ:

$$\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta) \tag{6}$$

Note that in this equation, m_e is the mass of the electron. Also from this, he derived the Compton wavelength of an electron, which takes the following form:

$$\lambda_c = \frac{\Delta\lambda}{\theta} = \frac{h}{m_e c}$$

Issue 4: The Double Slit Experiment

The next step came from Louis de Broglie in 1923, who suggested that waves not only had particle-like properties, but that all particles had wave-like properties. This is shown through the double-slit experiment.

For a photon, we can express momentum like so:

$$p = \frac{h\nu}{c} = \frac{h}{\lambda} \tag{7}$$

In this equation, ν is the wave's frequency and λ is the wave's length (wavelength). There is also another cool property called the wave vector $\vec{k}$, which is equal to:

$$\vec{k} = \frac{\vec{p}}{\hbar} = \frac{2\pi\vec{p}}{h} \tag{8}$$

De Broglie suggested that particles should act the same way, with:

$$\lambda = \frac{h}{p} = \frac{2\pi}{\vec{k}} \tag{9}$$

When experimentalists decided to test it, they setup a wall with two and sent a beam of electrons through to hit another wall further back. Classically, if particles did NOT look like waves, you would just get two peaks of electrons on the back wall. However, if particles actually act like waves, you get an interference pattern! Just like two ripples in a pond overlapping.

Figure 7: Water waves overlapping in a pond. Image credit to Linus Nylund on Unsplash.

When the experimentalists got together to test these things, they found that de Broglie's description was the right one. Therefore, not only is light both a particle and a wave, ALL matter is. Quite a lot to wrap your head around!

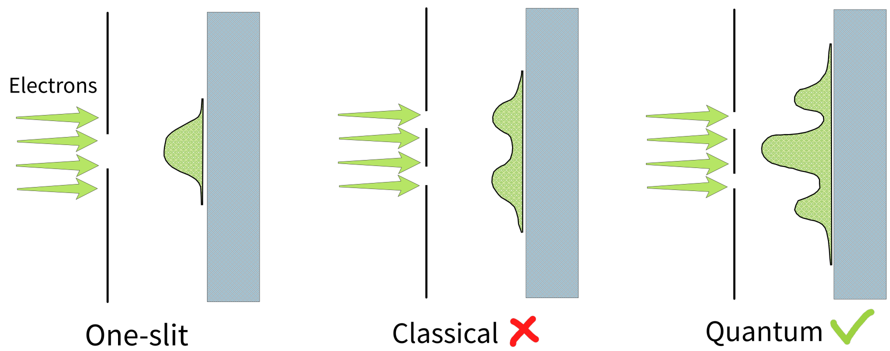

Figure 8: My attempt at drawing the double slit experiment. If you send electrons toward a wall with one slit, it will hit the back wall in this nice little bell curve. However, if you send the electrons through two slits, the waves will interfere with each other! You'll get an interference pattern on the back wall instead of just two different bell curves (note that I tried my best to draw it).

Issue 5: Heisenberg Uncertainty Principle

The last issue I'll talk about in this noteset comes straight off the heels of the "wave-particle duality" I described in the last section. When people really started to think about what de Broglie was saying, they realized it gave the biggest blow to classical physics yet: since waves are not "localized", or can't be pinned down to one location, neither is matter.

Classical physics can completely describe an object by its momentum and position: where it is and where it's going. Because you can determine these things, classical physics is *deterministic*.

However, when you get down to the scale of atoms, because everything is acting like a wave, there is an inherent uncertainty in both position and momentum. In 1925, Werner Heisenberg published the "Umdeutung" (reinterpretation) paper, which set up the basis for the *Heisenberg uncertainty principle* by saying that the position and momentum uncertainties were ACTUALLY related. The expression looks like this:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{10}$$

This says that the position uncertainty in the x-direction, multiplied by the monetum uncertainty in the x-direction, must be greater than or equal to the reduced Planck's constant ($\hbar = \frac{h}{2\pi}$) over two. Most importantly, it suggests that, unlike classical physics, you cannot completely know the present and future properties of a particle. Not only that, knowing one of these uncertainties to higher precision means that you will lose precision in the other. In other words, quantum physics, and the atomic world as a whole, is *underterministic*.

Conclusion and Future

Quantum mechanics deals with uncertainty by use of the wave function ($\psi(\vec{x}, t)$), which describes the amplitude of the de Broglie wave of a particle with respect to position and time. If you square the wave (note the || means we are dealing with complex components) like so: $|\psi(\vec{x}, t)|^2$, you get the intensity of the wave, which is equal to the probability that whatever particle you're studying will be in position $\vec{x}$ at time t. Following that logic, $|\psi(\vec{x}, t)|^2 d^3x$ is the probability that a particle will be found in some volume element d^3x at time t.

The rest of these ntoesets will focus heavily on the wave-function, since we can actually use it to say things about nature. Although quantum physics shows that the atomic world is undeterministic, strange, and completely against common intuition, we can use the power of the wave function to describe it.